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Optimization of two-stage transcritical carbon dioxide heat pump cycles

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Abstract

Optimization studies of two-stage transcritical carbon dioxide heat pump cycles, incorporating options such as flash gas bypass, flash intercooling and compressor intercooling, are presented based on cycle simulation. Sub-critical and super-critical thermodynamic and transport properties of carbon dioxide coded and then integrated with the simulation code for further analyses. Results exhibit improvement in performance by adopting optimal operating conditions. The optimum interstage pressure, thus obtained, deviate from the classical estimate of geometric mean of gas cooler and evaporator pressure. It is observed that the flash gas bypass system yields the best performance among the three two stage cycles analyzed. Internal heat exchanger effectiveness and compressor isentropic efficiency shows marginal influence on the system performance. Internal heat exchanger effectiveness shows marginal influence on the system performance while compressor isentropic efficiency shows an about 10% variation in *COP*. However, optimum gas cooler pressure and optimum intermediate pressure are only marginally affected. Based on the cycle simulations, correlations of optimum gas cooler pressure and inter-stage pressure in terms of gas cooler temperature and evaporator temperature are obtained. This would be useful as a guideline in design of such systems.

Keywords: CO_2 heat pump; Multistage; Inter-stage pressure; Transcritical cycle; Optimization

1. Introduction

Natural refrigerants, in general, and particularly CO_2 is increasingly becoming the refrigerant of choice to replace the environmentally harmful CFCs and HCFCs. CO_2 exhibits excellent heat transfer properties and is non-flammable and non-toxic. It has relatively lower specific volume, resulting in component size reduction for the same operating conditions.

Lorentzen and Pettersen [1–3] have shown in their seminal studies that difficulties connected with the low critical temperature of CO_2 (31.1 °C) can be successfully overcome by operating the system in the transcritical mode, where single-phase heat rejection occurs above the critical temperature in the gas cooler instead of condenser as in conventional systems, and where pressure and temperature can be controlled independently to obtain optimum performance. The gliding temperature in the gas cooler makes the CO_2 systems more economical for simultaneous cooling and heating applications. One of the major advantages of transcritical CO_2 heat pump systems is the high temperature lift compared to others.

The performance decline of CO_2 single stage systems at high heat rejection temperature can be effectively overcome by employing a two-stage or multistage system with an intercooler in between compression stages in parallel with the gas cooler. The selection of the intermediate pressure is an important parameter for a multistage system. Several researchers [4–6] have investigated the optimum inter-stage pressure on the basis of minimum work requirement. There is a fair agreement on the fact that the optimum intermediate pressure in a two-stage refrigeration system is quite close to the classical estimate, given by the geometric mean of gas cooler and evaporator pressure. Gupta and Prasad [7] optimized three stage refrigeration systems graphically for refrigerants R12, R22, and R714. They also developed the correlations to account for the effects of subcooling of condensate and super-heating of vapour in the evaporator each up to 15 K. It was concluded that staging is most beneficial for R714. Gupta [8] investigated a cascade refrigeration-heat pump system using R-22 in heat pump side and R-13 in the refrigera-

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Nomenclature

COP	coefficient of performance
h	enthalpy . . . . . . kJ kg^{-1}
$\dot{m}$	mass flow rate . . . . . . kg s^{-1}
P	pressure . . . . . . bar
q	specific heat transfer . . . . . . kJ kg^{-1}
R	regression correlation coefficient
T	temperature . . . . . . K
w	specific work . . . . . . kJ kg^{-1}

Greek symbols

ε	effectiveness
η	efficiency

Subscripts

1–9	refrigerant state points
2a, 5a, 4a	refrigerant state points
amb	ambient
cic	compressor intercooling
comp	compressor
ev	evaporator
fgb	flash gas bypass
fic	flash intercooling
gc	gas cooler
gm	geometric mean
ic	intercooler
iex	internal heat exchanger
is	isentropic
opt	optimum

tion side. Dhar and Arora [9] estimated the optimum interstage temperature for cascade systems numerically.

Since two-stage CO_2 refrigeration systems operate in a transcritical cycle, conclusions cited above and reported in the literature may not be exactly applicable and hence additional studies are surely needed to obtain specific results for such systems. Kim et al. [10] have presented a comprehensive review of two-stage transcritical CO_2 systems. Bell [11] has reported a theoretical investigation on two-stage parallel compression economization of a CO_2 refrigeration cycle. Although a few studies has been reported on the two-stage transcritical CO_2 cycle, optimization of such systems has not appeared in the open literature.

Interstage pressure is the other most critical parameter for optimizing *COP* in addition to gas cooler pressure in case of a multistage CO_2 refrigeration system. However, the concepts of perfect intercooling, as described for gas compressors, do not apply when dealing with refrigerant vapour. In the present work, a simulation code was developed for three types of two-stage CO_2 heat pump cycles to study the effect of operating and design parameters on system performance. For thermodynamic properties of carbon dioxide in both sub-critical and supercritical region, a subroutine CO2PROP [12] based on published correlations was used. Gas cooler pressure and intercooler pressure were simultaneously optimized. Correlations were obtained for optimum intermediate pressure and for optimum gas cooler pressure in terms of evaporation temperature and gas cooler exit temperature.

2. Two-stage transcritical CO_2 cycles

Two-stage cycles incorporating one of the three options such as *flash gas bypass*, *flash gas intercooling* and *compressor intercooling* have been simulated in this study. The flow diagrams and corresponding pressure–enthalpy diagrams developed using property code CO2PROP and the actual cycle representation on P–h plane for all three chosen systems are shown in Figs. 1–3. As shown in Fig. 1, the saturated vapour at state 1 is compressed to state 2a in the LP compressor and then mixed with saturated vapour at 3 from the flash chamber to attain state 4. The superheated vapour at 4 is then compressed in the HP compressor to 5a. The supercritical carbon dioxide at state 5a is cooled in the gas cooler to state 6, and then expanded in expansion cum float valve up to flash chamber pressure of state 7. The liquid gets separated in the flash chamber and is further expanded in an expansion device to evaporator pressure of state 9. The saturated vapour from flash chamber at state 3 goes to the HP compressor. Useful cooling is achieved in the evaporator by evaporating the CO_2 from 9 to 1. 1-2 and 3-4 are the isentropic compression processes, while 1-2a and 3-4a are the actual compression processes.

In the flash intercooling system, as shown in Fig. 2, the saturated vapour from the evaporator at state 1 is compressed in the LP compressor to state 2a when it enters the flash intercooler. De-superheating of the vapour takes place in the flash intercooler by evaporation of liquid CO_2. This increases the mass of CO_2 vapour to HP compressor. The saturated vapour from flash chamber at state 3 is compressed to state 4a and the supercritical vapour is cooled in the gas cooler to state 5. The vapour is then expanded in the expansion cum float valve to state 6. The liquid gets separated in the flash intercooler and is then further expanded in the expansion device to state 8 and eventually evaporates to state 1 producing cooling effect.

The two-stage CO_2 compression system with intercooling is shown in Fig. 3. The saturated vapour from the evaporator at state 1a is compressed to 2a in LP compressor and cooled to state 3 in the intercooler by external fluid. Ambient air is taken as the external fluid. Cooling of CO_2 is carried out in the intercooler in addition to gas cooler. The CO_2 vapour is further compressed to 4a in the HP compressor. The supercritical vapour at state 4a is cooled in the gas cooler to state 5. CO_2 vapour is further cooled in the internal heat exchanger to state 6. CO_2 further expands in the expansion device and enters the evaporator.

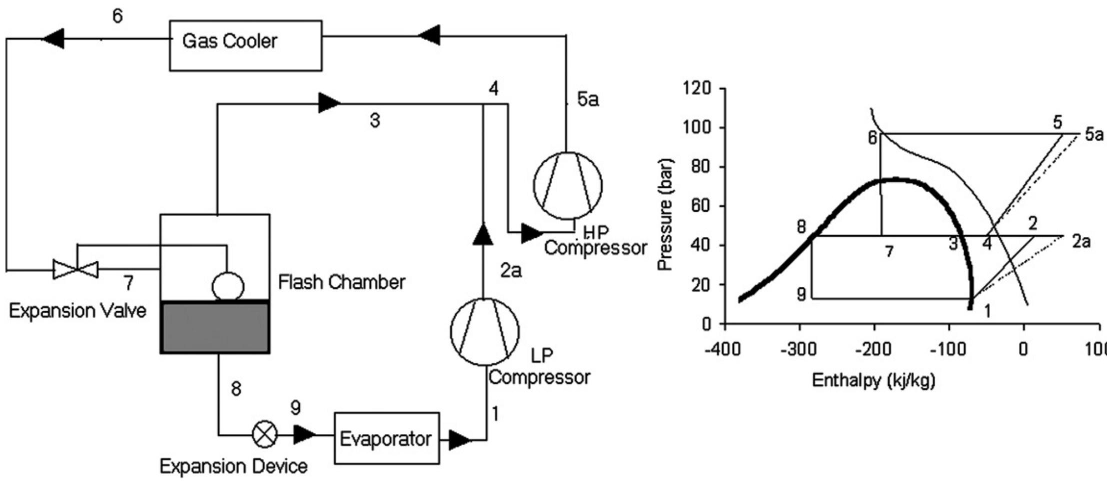

Fig. 1. Schematic and corresponding *P–h* diagrams for two-stage cycle with flash gas bypass.

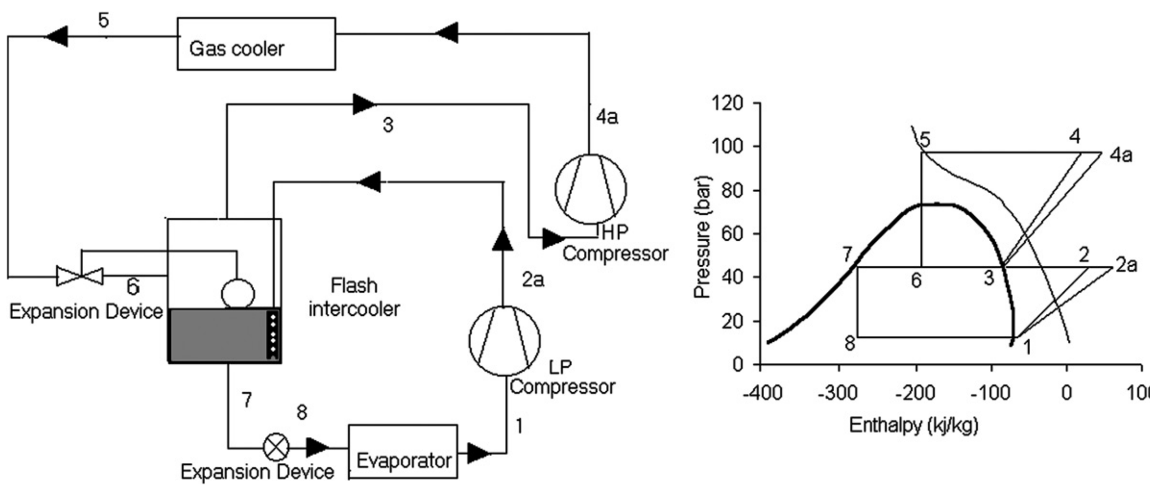

Fig. 2. Schematic and corresponding *P–h* diagrams for two-stage cycle with flash intercooling.

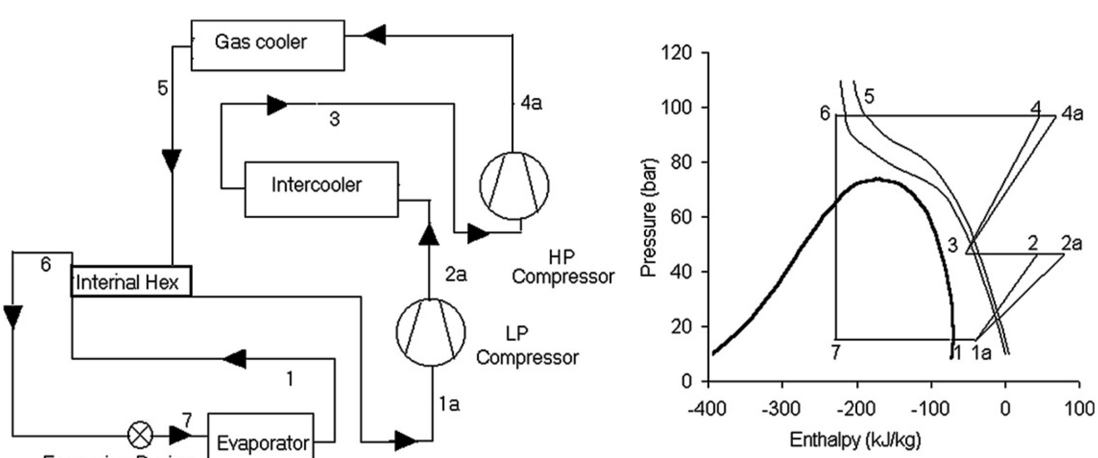

Fig. 3. Schematic and corresponding *P–h* diagram for two-stage cycle with compression intercooling.

3. Thermodynamic analysis

All the three systems have been modeled employing energy balance on individual components of the system. Steady flow energy equation and mass balance equation have been employed in each case. The following assumptions have been made to simplify the analysis:

(1) Heat transfer with the ambient is negligible.
(2) Single-phase heat transfer occurs for the external fluid.

(3) Compression process is adiabatic but non-isentropic.
(4) Evaporation, gas cooling and intercooling processes are isobaric.
(5) Vapour is at saturated condition at the exit to the flash intercooler and flash chamber and evaporator exit.
(6) State of the vapour is superheated at the exit to the intercooler.

Modular mathematical model is presented below:

i. *Mass and energy balance for flash chamber and flash intercooler*

(a) Flash gas bypass system

$$\dot m_3 + \dot m_8 = \dot m_7 \quad (1)$$

$$\dot m_8 = \dot m_9 = \dot m_1 = \dot m_{2a} = 1 \quad (2)$$

$$\dot m_3 + 1 = \dot m_7 \quad (3)$$

$$\dot m_4 = \dot m_{5a} = \dot m_6 = \dot m_7 \quad (4)$$

$$\dot m_3 h_3 + \dot m_8 h_8 = \dot m_7 h_7 \quad (5)$$

$$\dot m_3 = \frac{h_7 - h_8}{h_3 - h_7} \quad (6)$$

(b) Flash intercooler system

$$\dot m_{2a} + \dot m_6 = \dot m_7 + \dot m_3 \quad (7)$$

$$\dot m_7 = \dot m_8 = \dot m_1 = \dot m_{2a} = 1 \quad (8)$$

$$\dot m_3 = \dot m_{4a} = \dot m_5 = \dot m_6 \quad (9)$$

$$\dot m_{2a} h_{2a} + \dot m_6 h_6 = \dot m_7 h_7 + \dot m_3 h_3 \quad (10)$$

$$\dot m_3 = \frac{h_{2a} - h_7}{h_3 - h_6} \quad (11)$$

ii. *Energy balance in the internal heat exchanger*

$$h_{1a} - h_1 = h_5 - h_6 \quad (12)$$

iii. *Energy balance for the entire systems*

$$q_{ev} + w = q_{gc} \quad (13)$$

$$q_{ev} + w = q_{gc} + q_{ic} \quad (14)$$

(for compression with inter-cooling)

iv. *Refrigeration effect of evaporators*

$$q_{ev(fgb)} = h_1 - h_9 \quad (15)$$

$$q_{ev(fic)} = h_1 - h_8 \quad (16)$$

$$q_{ev(cic)} = h_1 - h_7 \quad (17)$$

v. *Heating effect of gas coolers and intercooler*

$$q_{gc(fgb)} = \dot m_3 (h_5 - h_6) \quad (18)$$

$$q_{gc(fic)} = \dot m_3 (h_4 - h_5) \quad (19)$$

$$q_{gc(cic)} = h_4 - h_5 \quad (20)$$

$$q_{ic(cic)} = h_{2a} - h_3 \quad (21)$$

vi. *Compressor work*

$$w_{(fgb)} = (h_{2a} - h_1) + \dot m_3 (h_{5a} - h_4) \quad (22)$$

$$w_{(fic)} = (h_{2a} - h_1) + \dot m_3 (h_{4a} - h_3) \quad (23)$$

$$w_{(cic)} = (h_{2a} - h_{1a}) + (h_{4a} - h_3) \quad (24)$$

vii. *Effectiveness of the internal heat exchanger and intercooler*

$$\varepsilon_{iex} = \frac{T_{1a} - T_1}{T_5 - T_1} \quad (25)$$

$$\varepsilon_{ic} = \frac{T_{2a} - T_3}{T_{2a} - T_{amb}} \quad (26)$$

viii. *Isentropic efficiency of compressors*

$$\eta_{(fgb,fic)is,comp} = \frac{h_2 - h_1}{h_{2a} - h_1} \quad (27)$$

$$\eta_{cic,is,comp} = \frac{h_2 - h_{1a}}{h_{2a} - h_{1a}} \quad (28)$$

ix. *System performance*

$$COP_{fgb,fic} = \frac{q_{gc} + q_{ev}}{w} \quad (29)$$

$$COP_{cic} = \frac{q_{gc} + q_{ev} + q_{ic}}{w} \quad (30)$$

4. Optimization

Previous studies [13,14] show that an optimum gas cooler pressure exists for the transcritical CO_2 cycle where it exhibits the maximum *COP* for a given cooler outlet temperature. This can be attributed to the unique behavioural pattern of CO_2 properties around the critical point and beyond, where the slope of the isotherms is quite modest for a specific pressure range; at pressures above and below this range, the isotherms become much steeper. However, in case of the two-stage CO_2 transcritical system, the intermediate pressure is also an influential parameter to decide the best *COP* along with the gas cooler pressure. While optimizing a two-stage CO_2 transcritical system with respect to optimum value of gas cooler pressure and intermediate pressure, it must be remembered that the optimum gas cooler pressure is functionally coupled with the intermediate pressure and with the mass flow rate at the second stage. This necessitates simultaneous optimization of the gas cooler pressure and intermediate pressure of the two stage CO_2 transcritical system. The optimum value of the gas cooler pressure is different from the corresponding value for a single stage cycle having a fixed evaporator and gas cooler outlet temperature as the optimum gas cooler pressure varies considerably with intermediate pressure.

In search of an optimum intermediate pressure in multistage vapour compression system, the temperature of the refrigerant at the beginning of compression in each stage cannot be the same, and, furthermore, since ideal gas laws do not apply, $\sqrt{P_{gc} P_{ev}}$ will not yield the optimum pressure ratio for each stage. However, $\sqrt{P_{gc} P_{ev}}$ may be used as a good initial guess, and the optimum pressure ratio for the individual stages must be ultimately established by an iterative solution [15].

5. Results and discussion

The transcritical CO_2 cycle is optimized on the basis of combined system *COP*, which is the sum of the heating and cooling mode *COP*s. All the three two stage systems considered in this study (flash gas bypass, flash gas intercooling and compressor intercooling) are simulated and their performance is evaluated on the basis of maximum combined *COP* to obtain the optimum gas cooler and intermediate pressures. These values are obtained for various operating conditions along with simultaneous variation of the compressor discharge pressure and intermediate pressure having a step size of 0.5 bar for each. The performance is evaluated on various evaporator temperature (−50 °C to −30 °C) and gas cooler outlet temperature (35 °C to 60 °C). The performance parameters with their optimum values are exhibited graphically as elucidated below.

Variation of maximum system *COP* with gas cooler outlet temperature for various evaporator temperatures is shown in Fig. 4. There is an increase of almost 35% in *COP* as the evaporator temperature increases from −50 °C to −30 °C. Variation is almost the same for all three systems. There is a sharp increase in *COP* for all the two-stage systems compared to a single stage system. However, for a given evaporator temperature, flash gas bypass system exhibits the highest *COP*. *COP* of the flash intercooling system is lower due to presence of additional mass in the second stage, which requires more work while compression intercooling system is having low *COP* due to low refrigeration effect as the flash gas which flows through the evaporator in this case does not yield any cooling effect. The flash gas bypass system yields better *COP* due to the fact that the vapour (which is separated in the separator) is made to *bypass* the evaporator where it would not have produced any cooling effect anyway and instead it is directed straight to the compressor at higher pressure thereby saving a bit of compressor power as well. However in intercooling systems additional mass of vapour is added to the second stage which increases the second stage compressor power pulling the *COP* down. The two-stage *COP* trend lines are steeper than those in a single stage system. As is experienced in single stage cycles [9], the cooler outlet temperature is an important parameter for the optimum design of multistage systems as well. Maximum system *COP* increases sharply with a decrease in gas cooler outlet temperature (Fig. 4).

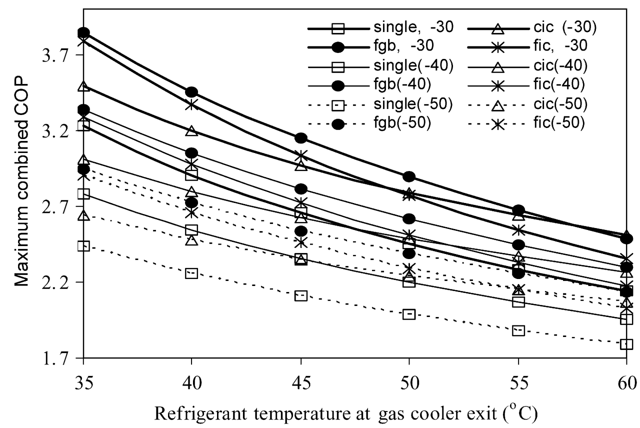

Fig. 4. Variation of maximum COP with gas cooler exit temperature.

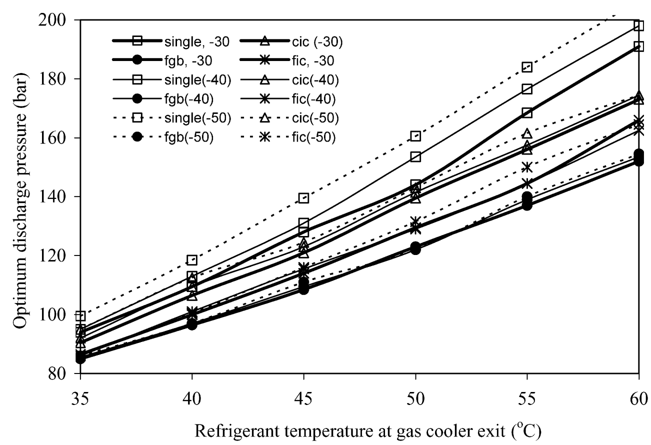

Fig. 5. Variation of optimum discharge pressure with gas cooler exit temperature.

As stated before, the only drawback associated with transcritical cycles is that the system operates at a very high discharge pressure. Moreover, due to the divergent nature of the isotherms in the supercritical region, the discharge pressure needs to be optimized to yield maximum *COP*. There is a sharp reduction in optimum discharge pressure by adopting staging in compression as shown in Fig. 5. The optimum discharge pressure for a flash gas bypass system is lowered by almost 30% at an evaporator temperature of −50 °C. The optimum discharge pressure turns out to be the lowest for the flash gas bypass system at a given evaporator temperature since intermediate pressure and discharge pressure, both are optimized simultaneously. Optimum discharge pressure also varies with evaporator temperature for a chosen multistage system. As the evaporator temperature decreases, the optimum discharge pressure increases. The ‘S’ shape and divergent nature of isotherms contribute to an increase in optimum discharge pressure at low evaporator and high gascooler temperatures. There is a marginal variation in optimum discharge pressure with evaporator pressure, whereas a significant rise is observed with increase in gas cooler exit temperature. Hence the flash gas bypass system is preferred over other two-stage systems owing to its better *COP* and lower optimum discharge pressure.

Discharge temperature has a significant effect on design and performance of the compressor. Any reduction in its value prolongs life of the compressor. As shown in Fig. 6, staging in the compression process brings down the discharge temperature considerably. The discharge temperature in a flash intercooling system is recorded to be the lowest because; this can be attributed to the large reduction in temperature occurring in the intercooler, whereby the refrigerant is brought down from superheated to saturated state at the intermediate stage. For an evaporator temperature of −50 °C, the discharge temperature gets reduced by almost one-third in a flash intercooling system compared to a single stage system while compressor intercooling brings the discharge temperature down by 15 °C which is only a 10% reduction in discharge temperature. Discharge temperature shows an increasing linear trend with almost an equal

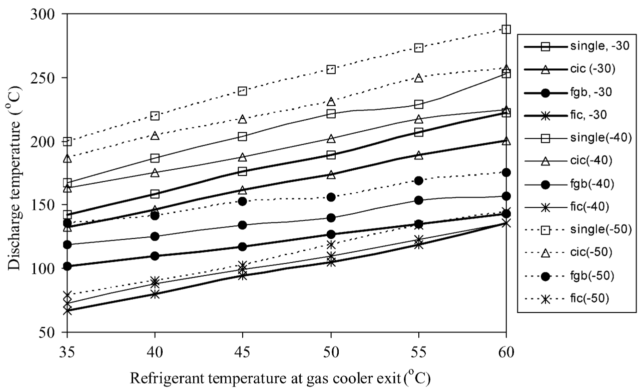

Fig. 6. Variation of compressor discharge temperature with gas cooler exit temperature.

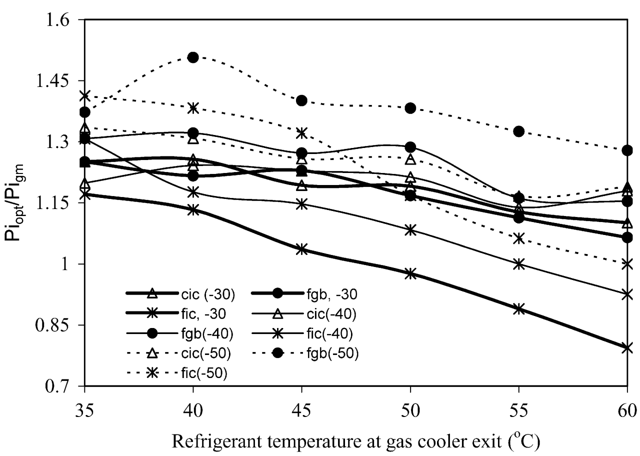

Fig. 7. Variation of optimum interstage pressure to geometric mean interstage pressure ratio with gas cooler exit temperature.

amount with gas cooler exit temperature for all evaporator temperatures.

As previously stated, interstage pressure plays an important role in the optimization of a two-stage transcritical CO_2 system. It is observed that there is considerable deviation in the optimum interstage pressure from the classical estimate, given by the geometric mean of gas cooler and evaporator pressure. The deviation increases as temperature lift increases as shown in Fig. 7. However, this deviation for compressor intercooling system is moderate compared to the other two systems. It is evident from Fig. 7 that the ratio of optimum intermediate pressure to geometric mean pressure becomes less than 1 at evaporator temperatures of −30 °C and −40 °C. The optimum pressure to geometric mean pressure ratio is almost the same for flash gas bypass system and compressor intercooling system at high evaporator temperature. As a result, the trend lines overlap each other quite often for these systems.

The comparison of optimum interstage pressure and geometric mean pressure of various systems at evaporator temperatures of −30 °C, −40 °C and −50 °C is presented in Figs. 8–10. It is evident that the optimum interstage pressure is higher than the geometric mean pressure at all the evaporator temperatures for flash gas bypass and compressor intercooling systems while it is lower for flash intercooling system as the gas cooler exit temperature increases. It is also noted from the figures that the optimum value of the interstage pressure of all the systems comes closer as the evaporator temperature decreases. Deviation of optimum interstage pressure from geometric mean pressure is relatively less for flash intercooling systems The geometric mean value show almost linear variation with gas cooler outlet temperature for a given evaporator temperature. However, it is not

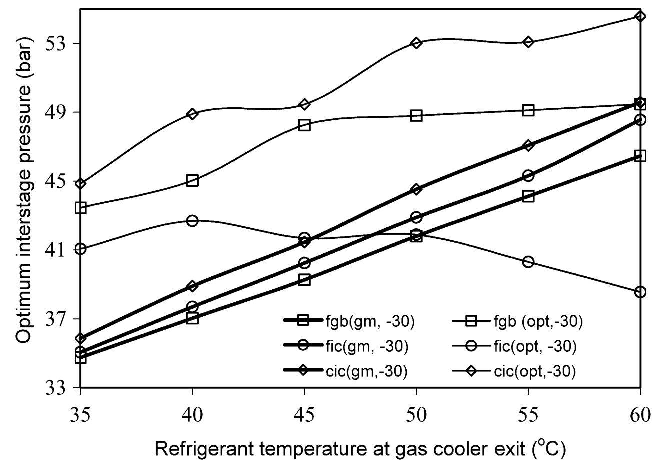

Fig. 8. Variation of interstage pressure with gas cooler exit temperature.

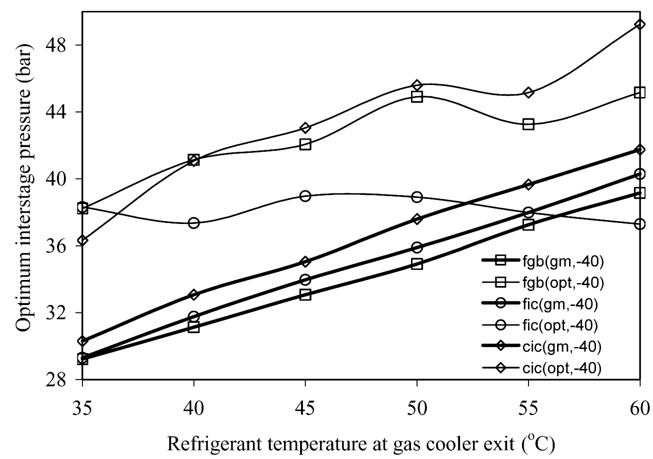

Fig. 9. Variation of interstage pressure with gas cooler exit temperature.

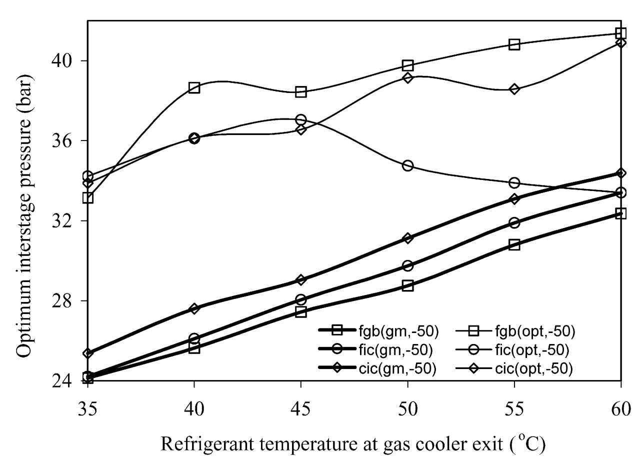

Fig. 10. Variation of interstage pressure with gas cooler exit temperature.

true for the optimum parameter. This could be attributed to the fact that the optimum interstage pressure occurs where the *COP* is maximum while the geometric mean pressure is calculated independently which does not involve any optimization procedure. It is evident from Figs. 8–10 that optimum interstage pressure decreases at lower evaporator temperature while optimum gas cooler pressure increases, as explained earlier, due to the unique shape of the isotherms for CO_2.

In the present study, the effects of compressor isentropic efficiency, internal heat exchanger effectiveness and intercooler effectiveness are also investigated. Compressor isentropic efficiency is considered within the range of 60%–80% while intercooler effectiveness taken within 0.6–0.7. It is observed that the effect of internal heat exchanger effectiveness, intercooler effectiveness and compressor isentropic efficiency on optimum gas cooler pressure and optimum interstage pressure is marginal, pretty much similar to what happens in a single stage CO_2 cycle. However, there is a significant variation in system *COP* as the compressor isentropic efficiency varies from sixty to eighty percent.

5.1. Correlation for optimum pressure

The maximum system *COP* and the corresponding optimum pressure is a function of evaporator temperature, compressor efficiency, gas cooler outlet temperature and heat exchanger effectiveness:

$$COP_{\max} = f(t_{ev}, t_{gc}, \eta_{is,comp}, \varepsilon)$$
$$P_{opt} = f(t_{ev}, t_{gc}, \eta_{is,comp}, \varepsilon) \tag{31}$$

However, as discussed earlier, the internal heat exchanger has a negligible effect on the system performance for given input temperatures. Compressor isentropic efficiency also has marginal effect on optimum pressures. Hence the functional optimum condition dependency can be simplified to:

$$COP_{\max} = f(t_{ev}, t_{gc}), \qquad P_{opt} = f(t_{ev}, t_{gc}) \tag{32}$$

The following correlations for optimum gas cooler pressure and optimum inter-stage pressure are obtained performing a thorough regression analysis on the data obtained from the system simulation.

$$P_{opt,gc(fgb)} = 25.11 - 0.087t_{ev} + (0.973 + 0.019t_{gc})t_{gc}$$
$$R^2 = 0.9985 \tag{33}$$

$$P_{opt,gc(fic)} = 16.94 - 0.08t_{ev} + (1.201 + 0.0201t_{gc})t_{gc}$$
$$R^2 = 0.9979 \tag{34}$$

$$P_{opt,gc(cic)} = -18.13 - 0.202t_{ev} + (2.741 + 0.006t_{gc})t_{gc}$$
$$R^2 = 0.9982 \tag{35}$$

$$P_{opt,ic(fgb)} = 20.68 + 0.421t_{ev} + (1.448 - 0.0128t_{gc})t_{gc}$$
$$R^2 = 0.9560 \tag{36}$$

$$P_{opt,ic(fic)} = 36.49 + 0.331t_{ev} + (0.705 - 0.008t_{gc})t_{gc}$$
$$R^2 = 0.9143 \tag{37}$$

$$P_{opt,ic(cic)} = 39.11 - 0.679t_{ev} + (0.973 + 0.0061t_{gc})t_{gc}$$
$$R^2 = 0.9566 \tag{38}$$

where R^2 = correlation coefficient which shows the goodness of fit in the regression analysis.

These correlations are valid for evaporation temperatures (t_{ev}) ranging between −50 °C and −30 °C and cooler exit temperatures (t_{gc}) ranging between 30 and 50 °C.

6. Conclusions

Thermodynamic analysis and performance simulation of two-stage transcritical CO_2 cycles have been presented here. Sub-critical and super-critical thermodynamic and transport properties of carbon dioxide were coded and then integrated with the system simulation code. The gas cooler pressure and interstage pressure are simultaneously optimized. It is found that for a two-stage transcritical carbon dioxide cycle an optimal heat rejection pressure and an optimum interstage pressure exist that yield a maximum system *COP*. The analysis reveals that staging not only enhances the performance, it improves the design as well as significantly brings down the optimum gas cooler pressure. It is observed that the flash gas bypass system yields the best performance among the three two-stage systems analyzed. Results also show that the deviation of optimum interstage pressure from the classical estimate, given by the geometric mean of the gas cooler and evaporator pressure increases as temperature lift increases. The deviation depends upon the gas cooler temperature, evaporator temperature and the system. Based on these cycle simulations, correlations for optimum gas cooler and interstage pressures has been obtained in terms of gas cooler temperature and evaporator temperature; these are expected to be of help in the design of such optimized systems.

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